Gauss' Law

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{encircle}}{\varepsilon_0}$$

Vaccum.

There are situations where we can use this Gauss' Law to get $\vec{E}$

This is possible when we can "guess" that $|\vec{E}|$ is constant on a surface.

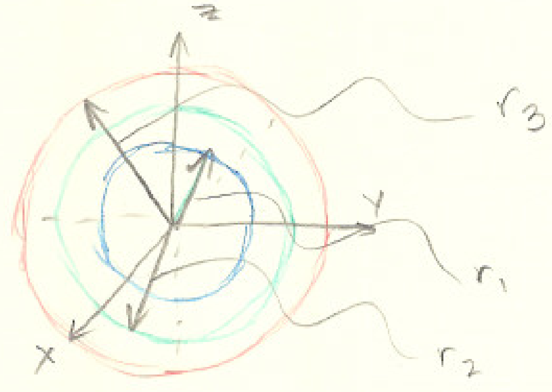
EX:

Sphere of constant volume charge density ρ_0

$$\int |\vec{E}(r_p)| \hat{r} \cdot |d\vec{s}| \hat{r} = \int |\vec{E}(r_p)| dS$$

$$= |\vec{E}(r_p)| \oint dS$$

$$= |\vec{E}(r_p)| 4\pi r_p^2$$



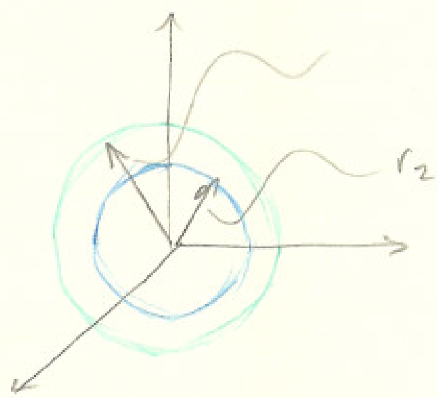
$$|\vec{E}(r_p)| = \frac{\frac{4\pi}{3} \rho_0 (r_2^3 - r_1^3)}{4\pi r_p^2 \varepsilon_0}$$

$$\vec{E}(r_p) = \frac{\rho_0}{3\varepsilon_0}\left(\frac{r_p^3 - r_1^3}{r_p^2}\right)\hat{r}_p$$

$$Q_{encl} = \int \rho \, dV$$ The ammount of charge enclosed.

$$= \rho_0 [\text{volume } r_2 - \text{volume } r_1]$$
$$= \rho_0 \left[\frac{4\pi r_2^3}{3} - \frac{4\pi r_1^3}{3}\right]$$
$$= \rho_0 \frac{4\pi}{3} (r_2^3 - r_1^3)$$

QED.



On the Gaussian Sphere of radiis r_p

$|\vec{E}(r_p)|$ is constant ; $|\vec{E}(r_p)|\hat{r}_p = \vec{E}(r_p)$

$d\vec{S}$ for a peice of gausian imaging sphere is:

$$d\vec{S} = dS\,\hat{r}_p$$

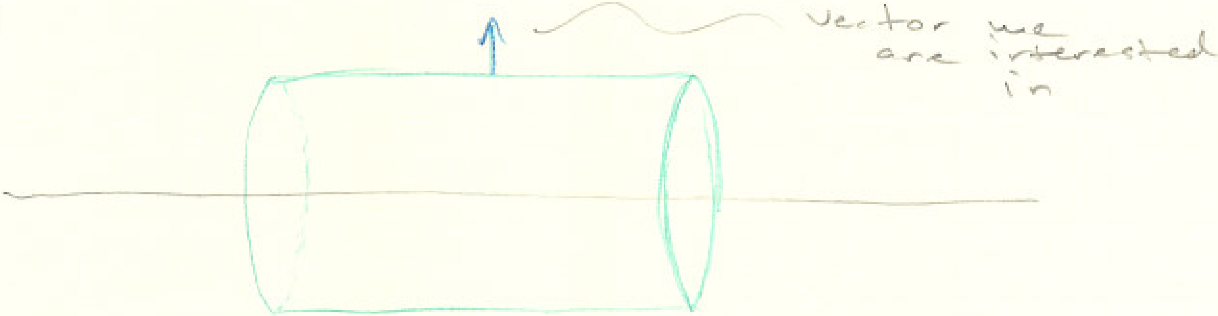
$$= r_p^2 \sin\theta \, d\theta \, d\phi$$

$$\oint \vec{E} \cdot d\vec{S} = |\vec{E}(r_p)| \, 4\pi r_p^2$$

$$= \frac{1}{\varepsilon_0} \int_0^{r_p} \rho_0 \, r^2 \, dr \, \sin\theta \, d\theta \, d\phi$$

$$= \frac{4\pi \rho_0 \, r_p^3}{3 \varepsilon_0}$$

EX:



$$\oint = \int_{\text{Front}} \vec{E} \cdot d\vec{S}_{\text{front}} + \int_{\text{Back}} \vec{E} \cdot d\vec{S}_{\text{back}} + \int_{\text{side}} \vec{E} \cdot d\vec{S}$$

The dot product of these two are ∅ b/c $\vec{E}$ is perpendiculare to the surface.

$$\vec{E}(r_p) = |\vec{E}(r_p)| \, \hat{r}_p$$

$$d\vec{S} = r_p \, d\phi \, dz \, \hat{r}_p$$

$$\vec{E}\cdot d\vec{s} = |\vec{E}(r_p)|\,\hat{r}_p \cdot (r_p\, d\phi\, dz\, \hat{r}_p)$$

$$= |\vec{E}(r_p)|\, r_p\, d\phi\, dz$$

$$\oint \vec{E}\cdot d\vec{s} = |\vec{E}(r_p)|\, r_p\, 2\pi l$$

$$Q_{encl} = \lambda_0 l$$

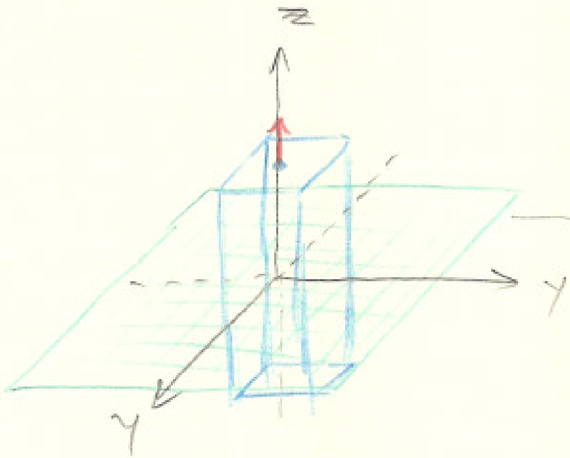
$$|\vec{E}(r_p)|\, r_p\, 2\pi l = \frac{\lambda_0 l}{\varepsilon_0}$$

$$|\vec{E}(r_p)| = \frac{\lambda_0}{2\pi\varepsilon_0}\,\frac{1}{r_p}$$

$$\vec{E}(\vec{r}_p) = \frac{\lambda_0}{2\pi\varepsilon_0}\,\frac{1}{r_p}\,\hat{r}_p$$

Here we have solved the problem w/out complicated integration.

EX:



∞ plane on the xy plane w constant surface charge density

We can see that there will be no effect of the electric feild of the sides; b/c there are at right angles to the point in question

$$|E(z_p)| = |E(-z_p)|$$

$$\oint \vec{E} \cdot d\vec{S} = |\vec{E}(z_p)| \cdot \text{Area} + |\vec{E}(-z_p)| \text{Area}$$

$$= 2|\vec{E}(z_p)| \text{Area}$$

$$= \frac{1}{\varepsilon_0} \sigma_0 \text{ Area}$$

$$|\vec{E}(z_p)| = \frac{\sigma_0}{2\varepsilon_0}$$

$$\vec{E} = \frac{\pm \sigma_0 \hat{z}}{2\varepsilon_0}$$

note: There will be a midterm & final question using one of these 3 types of gaussian simplification.